Joint Optimization of Spectrum Sensing Time and Threshold in Multichannel Cognitive Radio

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***Abstract*—In order to improve the throughput of multichannel CR, a joint optimization of spectrum sensing time and sub channel sensing threshold based on the alternating direction optimization is proposed. Based on the SU's listen-before-transmit frame structure, an optimized allocation model is built to maximize the aggregate throughput of the SU over all the sub channels, providing that the communication demand of the PU and the performance of the sub channel spectrum sensing are guaranteed. The joint optimization algorithm is proposed to obtain the optimal solutions to the model through alternatively optimizing sensing threshold and time. The simulation results show that there exits the optimal sensing time and threshold that maximize the SU's throughput, and compared to the previous schemes, the joint allocation can improve the SU's throughput.**

Keywords*—cognitive radio; spectrum sensing; joint optimization; throughput.

I. INTRODUCTION

Based on the conventional fixed spectrum allocation policy, most available radio spectra have been assigned to registered users, which lead to a serious waste of spectrum utilization. In fact, recent reports from Federal Communications Commission (FCC) have shown that only 30% of the allocated spectrum in US is fully utilized [1]. Cognitive radio, which enables secondary users to utilize the spectrum when primary users are not occupying it, has been proposed as a promising technology to improve spectrum utilization efficiency [2–4], and has three essential components: (1) Spectrum sensing: the secondary users sense the radio spectrum environment within their operating range to detect the frequency bands which are not occupied by primary users; (2) Dynamic spectrum management: cognitive radio networks dynamically select the best available bands for communication; (3) Adaptive communications: a cognitive radio device can configure its transmission parameters (e.g., carrier frequency, transmission power) to opportunistically make best use of the ever changing available spectrum [5].

Spectrum sensing is a fundamental task for cognitive radio, and the most common method for spectrum sensing is energy detection, which does not require any primary signal information, and it is robust to unknown dispersed channels and fading. Based on the SU's listen-before-transmit frame structure, at the beginning of each frame, the SU senses the PU, if the PU is not detected, the SU starts to transmit the data. Multichannel CR allows the SUs to use multiple idle sub channels simultaneously to improve the overall throughput. The performance of the CR is determined by sensing time and sensing threshold [5-6], the longer sensing time the better sensing performance, but the longer sensing time means the shorter transmit time for SU, and smaller detection threshold will increase the probability of detection, but it also increase the probability of the false alarm and decrease the spectrum utilization of SU. So, the appropriate sensing time and threshold is important for increasing the performance of CR. In [7], the authors optimized the sensing threshold, providing that the increase of the throughput of SU and the performance of spectrum sensing is guaranteed. In [5], the authors designed the sensing slot duration to maximize the achievable throughput for the SUs under the constraint that the PUs are sufficiently protected with a fixed sensing threshold.

In this paper, we propose a joint optimization algorithm to obtain the optimal solutions to the model through alternatively optimizing sensing threshold and time. The simulation results show that there exits the optimal sensing time and threshold to maximize the SU's throughput, compared to the previous schemes, the joint allocation can improve the SU's throughput.

This paper is organized as follows. Section II presents the model of SU's throughput and reviews the energy detection scheme. In Section III optimization of sensing time and threshold are proposed. Simulation results are given in Section IV. Conclusions are finally drawn in Section V.

II. ENERGY DETECTION AND SYSTEM MODEL

Suppose that we are interested in the frequency band with the number of sub channels is L, the discrete received signal at the secondary user in sub channel l can be represented as

$$y_l(t)=\begin{cases} n_l(t), H_l^0 \\ h_l s_l(t)+n_l(t), H_l^1 \end{cases} \quad (1)$$
$$t=(1,2,3,\dots M)$$

Where t is the sample sequence, s_l is the PU signal with power p_s^l, n_l is the white Gaussian noise with power σ_l^2, h_l is the channel gain on sub channel l between PU and SU, M is number of samples, H_l^1 is the hypothesis that the PU is active, and the H_l^0 is the hypothesis that the PU is inactive. Let τ be the sensing time and f_s be the sampling frequency, the number of samples can be represented as

$$M = \tau f_s \tag{2}$$

The test statistic for energy detector is given by

$$Y_l = \frac{1}{M}\sum_{t=1}^{M}\left|y_l(t)\right|^2 \tag{3}$$

Where $y_l(1), y_l(2), \ldots y_l(M)$ are independent and identically distributed random process, If M is large enough, we use Gaussian approximations to the probability density functions of the test statistic Y_l

$$Y_l = \begin{cases} N\left(\sigma_l^2, \dfrac{1}{M}\sigma_l^4\right) \\ N\left((1+\gamma_l)\sigma_l^2, \dfrac{(1+2\gamma_l)}{M}\sigma_l^4\right) \end{cases} \tag{4}$$

$N(a,b)$ is a Gaussian independent and identically distributed (iid) random process with mean a and variance b, $\gamma_l = h_l^2 p_s^l / \sigma_l^2$ is the signal to noise ratio of spectrum sensing on sub channel l.

If we set the detection threshold as ε_l, the probability of false alarm and the probability of detection are given by

$$\begin{cases} P_l^f(\tau,\varepsilon_l) = Q\left(\dfrac{\varepsilon_l - \sigma_l^2}{\sqrt{\sigma_l^4/(\tau f_s)}}\right) \\ P_l^d(\tau,\varepsilon_l) = Q\left(\dfrac{\varepsilon_l - (1+\gamma_l)\sigma_l^2}{\sqrt{(1+2\gamma_l)\sigma_l^4/(\tau f_s)}}\right) \end{cases} \tag{5}$$

Where $Q(\bullet)$ is the complementary distribution function of the standard Gaussian,

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{t^2}{2}\right)dt \tag{6}$$

Fig.1 shows the listen-before-transmit frame structure designed for a cognitive radio network with where each frame consists of one sensing slot and one data transmission slot.

Suppose the sensing duration is τ, and the frame duration is T. Denote $c_l^0 = \log_2\left(1+\gamma_l^{SU}\right)$ as the throughput of the SU when it operates in the absence of PU, and $c_l^1 = \log_2\left(1+\dfrac{\gamma_l^{SU}}{1+\gamma_l}\right)$ is the throughput when it operates in the presence of primary users. Where γ_l^{SU} is the signal to noise ratio of SU on sub channel l.

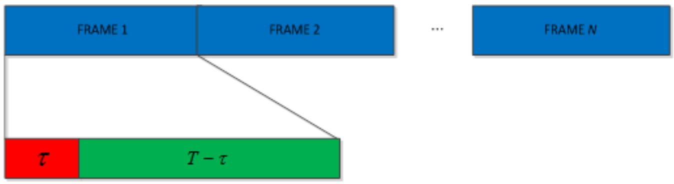

Fig.1. listen-before-transmit frame structure for cognitive radio networks

For a given frequency band of interest, let us define $P(H_l^1)$ as the probability for which the primary user is active, and $P(H_l^0)$ as the probability for which the primary user is inactive, and $P(H_l^1)+P(H_l^0)=1$. The aggregate throughput of the SU over all the sub channels is given by

$$\begin{aligned} C(\tau,\varepsilon) = \left(\frac{T-\tau}{T}\right)\sum_{l=1}^{L}\Big(P(H_l^0)c_l^0\left(1-P_l^f(\tau,\varepsilon_l)\right) \\ +P(H_l^1)c_l^1\left(1-P_l^d(\tau,\varepsilon_l)\right)\Big) \end{aligned} \tag{7}$$

We denote r_l^0 as the throughput of the SU when it operates in the absence of PU, and r_l^1 as the throughput when it operates in the presence of primary users. Then $r_l^0 = \log_2\left(1+\gamma_l^{PU}\right)$ and $r_l^1 = \log_2\left(1+\dfrac{\gamma_l^{PU}}{1+\gamma^{PS}}\right)$, where γ_l^{PU} is the signal to noise ratio of PU, and γ^{PS} is the signal to noise ratio from SU to PU. Then the aggregate throughput of the PU over all the sub channels is given by

$$\begin{aligned} R(\tau,\varepsilon) = \frac{P(H_l^1)}{T}\tau\sum_{l=1}^{L}r_l^0 \\ +(T-\tau)\sum_{l=1}^{L}\left(r_l^0 P_l^d(\tau,\varepsilon_l)+r_l^1\left(1-P_l^d(\tau,\varepsilon_l)\right)\right) \end{aligned} \tag{8}$$

III. OPTIMIZATION OF OVERALL THROUGHPUT OF SU

In this paper we propose a joint optimization algorithm to maximize the throughput of the SUs through alternatively optimizing sensing threshold and time, providing that the communication demand of the PU and the performance of the sub channel spectrum sensing are guaranteed. The optimization model can be represented by

$$\begin{aligned} &\max_{\tau,\varepsilon} C(\tau,\varepsilon) \\ &s.t.\begin{cases} R(\tau,\varepsilon) \ge \xi, \\ P_l^f(\tau,\varepsilon_l) \le \alpha, \\ P_l^d(\tau,\varepsilon_l) \ge \beta, l = 1,2,\cdots L. \end{cases} \end{aligned} \tag{9}$$

Where $\beta \ge 0.5$ is the target probability of detection which the PU are defined as being sufficiently protected, $\alpha \le 0.5$ is the maximal probability of false detection which the SU can sufficiently use the sub channel l and the $R(\tau,\varepsilon)$ is the minimal throughput required by the PU. Equation (6) is a multiple variable optimization problem, and we use alternative optimization method to solve this problem. Firstly, we set an initiate value for sensing time τ, then we search an optimal sensing threshold ε^* to maximize the C. Secondly, we set $\varepsilon = \varepsilon^*$ to search an optimal sensing time τ^*. After some iterations, we will find the global optimal value of sensing time and sensing threshold to maximize the C.

A. Optimization of Sensing Threshold

We set the sensing time $\tau = \tau_0$ and substituting (9) into (3) gives:

$$\begin{aligned} &\varepsilon_l^{\min} < \varepsilon_l < \varepsilon_l^{\max} \\ &\varepsilon_{\min} = \left(\frac{Q^{-1}(\alpha)}{\sqrt{\tau_0 f_s}} + 1 \right) \sigma_l^2 \\ &\varepsilon_{\max} = \left(Q^{-1}(\beta) \sqrt{\frac{2\gamma_l + 1}{\tau_0 f_s}} + \gamma_l + 1 \right) \sigma_l^2 \end{aligned} \tag{10}$$

Substituting $R(\tau,\varepsilon) \geq \xi$ into (8) gives: $\sum_{l=1}^{L} \Delta r_l P_l^d = g(\tau_0)$ where

$$\begin{aligned} &\Delta r_l = r_l^0 - r_l^1 \\ &g(\tau_0) = \frac{\xi}{\eta_l^1 (T - \tau_0)} - \frac{\tau_0}{T - \tau_0} \sum_{l=1}^{L} r_l^0 - \sum_{l=1}^{L} r_l^1 \end{aligned} \tag{11}$$

Thus, (9) can be converted to

$$\begin{aligned} \max_{\varepsilon} C(\varepsilon) = &\left(\frac{T - \tau_0}{T} \right) \sum_{l=1}^{L} P(H_l^0) c_l^0 \left(1 - P_l^f(\varepsilon_l)\right) \\ &+ P(H_l^1) c_l^1 \left(1 - P_l^d(\varepsilon_l)\right) \\ s.t. &\begin{cases} \sum_{l=1}^{L} \Delta r_l P_l^d(\varepsilon_l) \geq g(\tau_0) \\ \varepsilon_l^{\min} < \varepsilon_l < \varepsilon_l^{\max} \ (l = 1, 2, \ldots L) \end{cases} \end{aligned} \tag{12}$$

For equality constraints in (12), they can be modified to unconstrained by integrating positive Lagrange multiplier, leading to:

$$\begin{aligned} L(\varepsilon) = &\sum_{l=1}^{L} \left(P(H_l^0) c_l^0 \left(1 - P_l^f(\varepsilon_l)\right) + P(H_l^1) c_l^1 \left(1 - P_l^d(\varepsilon_l)\right) \right) \\ &+ \lambda \left(\sum_{l=1}^{L} \Delta r_l P_l^d(\varepsilon_l) - g(\tau_0) \right) \end{aligned} \tag{13}$$

To maximize $L(\varepsilon)$ by requiring the $\frac{\partial L(\varepsilon)}{\partial \varepsilon} = 0$, thus the threshold could be given by

$$\begin{aligned} \varepsilon_l^0 = &\left(\sqrt{\frac{1}{4} + \frac{1}{2}\gamma_l + \frac{(1+2\gamma_l)}{(\tau_0 f_s)\gamma_l} \ln\left(\frac{P(H_l^0) c_l^0 \sqrt{(1+2\gamma_l)}}{\lambda \Delta r_l - P(H_l^1) c_l^1} \right)} \right. \\ &\left. + \frac{1}{2} \right) \sigma_l^2 \end{aligned} \tag{14}$$

Substituting (14) into $\sum_{l=1}^{L} \Delta r_l P_l^d(\varepsilon_l) = g(\tau_0)$ we can obtain the value of λ. The optimal value of sensing threshold could be given by

$$\begin{aligned} &\varepsilon_l^* = \begin{cases} \varepsilon_l^{\min}, \varepsilon_l^0 < \varepsilon_l^{\min} \\ \varepsilon_l^0, \varepsilon_l^{\min} < \varepsilon_l^0 < \varepsilon_l^{\max} \\ \varepsilon_l^{\max}, \varepsilon_l^0 > \varepsilon_l^{\max} \end{cases} \\ &(l = 1, 2, \ldots L) \end{aligned} \tag{15}$$

B. Optimization of Sensing Time

In section III.A we set the initial sensing time as $\tau = \tau_0$ to obtain the optimal sensing threshold $\varepsilon^* = [\varepsilon_1^*, \varepsilon_1^*, \ldots \varepsilon_L^*]$, the associate probability of detection is $P_l^d(\tau_0, \varepsilon_l^*)$. In this section, we fix the sensing threshold as $\varepsilon = \varepsilon^*$, then search the optimal sensing time $\tau = \tau^*$ to maximize the throughput C. Based on (5) the probability of false alarm is represented as

$$P_l^f = Q\left(Q^{-1}(P_l^d) \sqrt{1 + 2\gamma_l} + \gamma_l \sqrt{\tau f_s} \right) \tag{16}$$

To ensure $R(\tau,\varepsilon) \geq \xi$ and $P_d^f(\tau, \varepsilon_l) \geq \beta$, it is obviously that $P_l^d(\tau, \varepsilon_l) > P_l^d(\tau_0, \varepsilon_l^*)$, since $Q(x)$ is a monotonically decreasing function of x, from (16), $P_l^f(\tau, \varepsilon_l) \geq P_l^f(\tau_0, \varepsilon_l^*)$, from (7) $C(\tau,\varepsilon) \leq C(\tau_0, \varepsilon^*)$. So, if $P_l^d(\tau, \varepsilon_l) = P_l^d(\tau_0, \varepsilon_l^*) = \mathrm{P}_l$ throughput of SU C will be maximal. Substituting (14) and $P_l^d = \mathrm{P}_l$ into (9) it gives

$$\begin{aligned} &\max_{\tau} C(\tau) = \left(\frac{T - \tau}{T} \right) \sum_{l=1}^{L} P(H_l^0) c_l^0 (1 - \\ &Q\left(Q^{-1}(\mathrm{P}_l) \sqrt{1 + 2\gamma_l} + \gamma_l \sqrt{\tau f_s} \right)) + P(H_l^1) c_l^1 (1 - \mathrm{P}_l) \\ &s.t. P_l^f(\tau, \varepsilon_l) \leq \alpha, l = 1, 2, \cdots L. \end{aligned} \tag{17}$$

Substituting $P_l^d = \mathrm{P}_l$ into $s.t. P_l^f(\tau, \varepsilon_l) \leq \alpha, l = 1, 2, \cdots L.$ it gives $\tau \geq \max(\tau_1, \tau_2, \ldots \tau_L)$ and (17) can be converted to

$$\begin{aligned} &\max_{\tau} C(\tau) = \left(\frac{T - \tau}{T} \right) \sum_{l=1}^{L} P(H_l^0) c_l^0 (1 - \\ &Q\left(Q^{-1}(\mathrm{P}_l) \sqrt{1 + 2\gamma_l} + \gamma_l \sqrt{\tau f_s} \right)) + P(H_l^1) c_l^1 (1 - \mathrm{P}_l) \\ &s.t. \tau \geq \max(\tau_1, \tau_2, \ldots \tau_L). \end{aligned} \tag{18}$$

Firstly, we must proof that when $\tau \in [0, T]$ $C(\tau)$ is a convex function of τ. $C(\tau)$ can be converted to

$$\begin{aligned} C(\tau) = &\left(\frac{T - \tau}{T} \right) \sum_{l=1}^{L} (P(H_l^0) c_l^0 \left(1 - P_l^f(\tau)\right) \\ &+ P(H_l^1) c_l^1 (1 - \mathrm{P}_l)) \end{aligned} \tag{19}$$

$\nabla^2 C(\tau)$ can be represented by

$$\begin{aligned}\nabla^2 C(\tau) &= 2P\left(H_l^0\right)c_l^0 \nabla P_l^f(\tau) \\ &\quad -(T-\tau)P\left(H_l^0\right)c_l^0 \nabla^2 P_l^f(\tau)\end{aligned} \tag{20}$$

The first derivative of $P_l^f(\tau)$ is negative and second derivative of $P_l^f(\tau)$ is positive, so $\nabla^2 C(\tau) < 0$ and the $C(\tau_{\max})$ will be maximal if $\nabla C(\tau_{\max}) = 0$. The optimal sensing time is

$$\tau^* = \max\left(\tau_{\max}, \max\left(\tau_1, \tau_2, \ldots \tau_L\right)\right) \tag{21}$$

C. *Joint Optimization of Sensing Time and Sensing Threshold*

The process of joint optimization of sensing time and threshold is shown as below:

1) Set the initial parameter: $k=1$, $\tau^k = 0, \varepsilon^k = \{0\}$, and estimation error ξ;

2) Set $\tau^k = \tau_0, \tau_0 \in [0, T]$;

3) Substituting τ_0 into (12) to obtain the ε^*;

4) Set $\varepsilon^{k+1} = \varepsilon^*$;

5) Substituting $\varepsilon^{k+1} = \varepsilon^*$ into (15) to obtain the τ^*;

6) Set $\tau^{k+1} = \tau^*, k = k+1$;

7) Repeat 3) ~ 6) until $\left|\tau^k - \tau^{k-1}\right| \le \xi, \left|\varepsilon^k - \varepsilon^{k-1}\right| \le \xi$.

IV. Results and Discussion

In simulations, we set $T = 1s, \sigma_l^2 = 1mW$, $L = 10, \alpha = 0.5$, $\beta = 0.9$ and $P\left(H_l^0\right) = P\left(H_l^1\right) = 0.5$. The transmit power of SU and PU is $10mW$, the multi-channel gain is Rayleigh distribution with mean $-10dB$, and estimation error $\xi = 0.01$.

In Fig.2 we show the achievable throughput versus the sensing time allocated to each frame for the secondary network. It is seen that there exist a sensing time to maximize the throughput of SU. In Fig.3 we show the achievable throughput versus the average sensing threshold allocated to all sub channels for the secondary network. It is also seen that there exist a sensing threshold to maximize the throughput of SU.

In Fig.4, we compare the throughput of SU and PU, in this figure four results are shown: fixed sensing threshold and time, optimization of sensing time, compromise of sensing time and threshold, joint optimization of sensing time and threshold. It is seen that, the proposed joint optimization of sensing time and threshold leads to more throughput than other three methods.

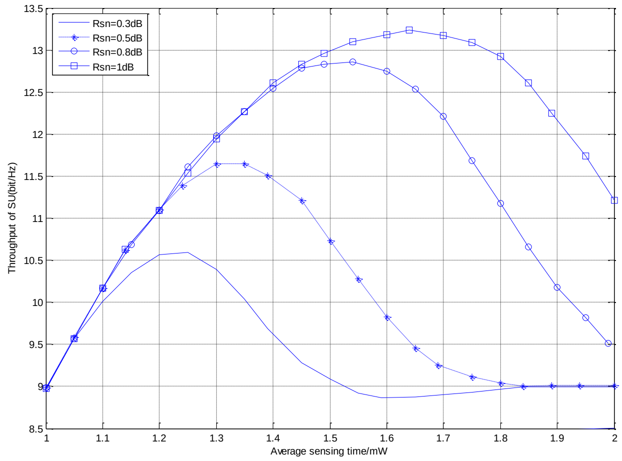

Fig.2. SU throughput versus sensing time

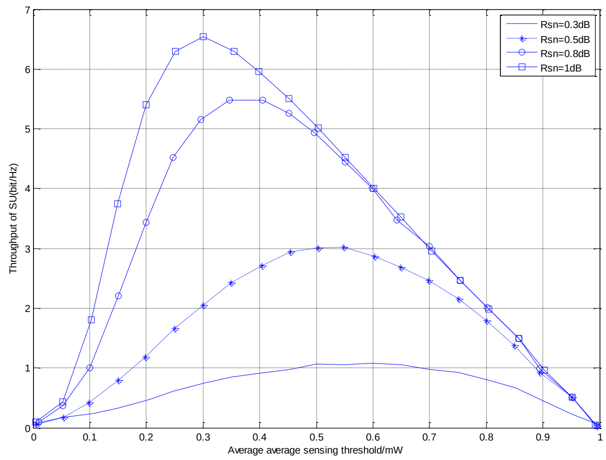

Fig.3. SU throughput versus average sensing threshold

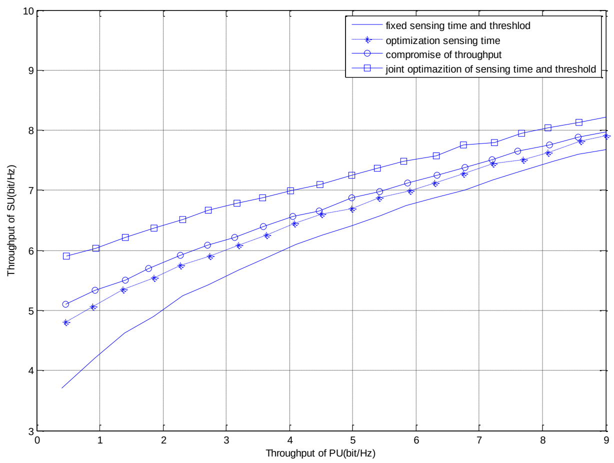

Fig.4. SU throughput versus PU throughput

V. Conclusions

Based on the SU's listen-before-transmit frame structure, a joint optimization of spectrum sensing time and threshold model to maximize the aggregate throughput of the SU over all the sub channels is proposed in this paper. The joint optimization algorithm alternatively optimizes sensing threshold and time to obtain the optimal solutions to the model.

The proposed method actually is optimization problems for sensing time and sensing threshold respectively. Theoretical analysis and simulation results show that these optimization problems have a joint global optimal solution to maximize the throughput of SU. At a given throughput of PU, the proposed joint optimization method will obtain more throughput of SU.

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